

## WEIGHTED BOUNDEDNESS OF THE HARDY–LITTLEWOOD MAXIMAL AND CALDERÓN–ZYGmund OPERATORS ON ORLICZ–MORREY AND WEAK ORLICZ–MORREY SPACES

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*Abstract.* For the Hardy–Littlewood maximal and Calderón–Zygmund operators, the weighted boundedness on the Lebesgue spaces are well known. We extend these to the Orlicz–Morrey spaces. Moreover, we prove the weighted boundedness on the weak Orlicz–Morrey spaces. To do this we show the weak–weak modular inequality. The Orlicz–Morrey space and its weak version contain weighted Orlicz, Morrey and Lebesgue spaces and their weak versions as special cases. Then we also get the boundedness for these function spaces as corollaries.

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