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A SIMPLE COUNTEREXAMPLE FOR THE PERMANENT–ON–TOP CONJECTURE

Hoang Anh Tran

Abstract. The permanent-on-top conjecture (POT) was an important conjecture on the largest eigenvalue of the Schur power matrix of a positive semi-definite Hermitian matrix, formulated by Soules. The conjecture claimed that for any positive semi-definite Hermitian matrix H, $\operatorname{per}(H)$ is the largest eigenvalue of the Schur power matrix of the matrix H. After half a century, the POT conjecture has been proven false by the existence of counterexamples which are checked with the help of computer. It raises concerns about a counterexample that can be checked by hand (without the need of computers). A new simple counterexample for the permanent-on-top conjecture is presented which is a complex matrix of dimension 5 and rank 2.

Mathematics subject classification (2020): 20G05, 15A15, 15A18, 15A03, 47A80.

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