

BOUNDEDNESS OF RIEMANN–LIOUVILLE OPERATOR FROM WEIGHTED SOBOLEV SPACE TO WEIGHTED LEBESGUE SPACE FOR $1 < q < p < \infty$

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Abstract. In the paper, a criterion for the boundedness of the Riemann-Liouville fractional integration operator from a weighted Sobolev space to a weighted Lebesgue space is obtained for $1 < q < p < \infty$.

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