

GEOMETRIC HARDY INEQUALITIES VIA INTEGRATION ON FLOWS

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Abstract. We introduce a geometric approach of integration along integral curves for functional inequalities involving directional derivatives in the general context of differentiable manifolds that are equipped with a volume form. We focus on Hardy-type inequalities and the explicit optimal Hardy potentials that are induced by this method. We then apply the method to retrieve some known inequalities and establish some new ones.

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