

MAJORIZATION REFINEMENTS OF KY FAN'S EIGENVALUE INEQUALITY AND RELATED RESULTS

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Abstract. In this note we show the anti-isotonicity of a certain operator function induced by the eigenvalue map $\lambda(\cdot)$ on the space of Hermitian matrices. In consequence, we obtain some majorization refinements of the Ky Fan's eigenvalue inequality. Thus we extend Maligranda's inequalities from a norm to the eigenvalue map, and from the usual order on \mathbb{R} to the majorization preorder on \mathbb{R}^n .

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