

PRE-GRÜSS AND GRÜSS–OSTROWSKI LIKE INEQUALITIES IN BANACH SPACES

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Abstract. For a given Banach space and its dual space we investigate a Chebyshev type functional. We derive a pre-Grüss inequality for the functional. We discuss various variants of assumptions leading to this inequality. To do so, we employ some superquadratic as well as convex control functions in order to weaken the classical Dragomir's condition. Next, we establish a corresponding Grüss-Ostrowski like inequalities for the space $L^p_{[a,b]}$.

Mathematics subject classification (2020): 46B10, 46B20, 26D10, 26D15.

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