

LATTICE EMBEDDINGS IN FREE BANACH LATTICES OVER LATTICES

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Abstract. Any lattice embedding $i: \mathbb{L} \rightarrow \mathbb{M}$ between two lattices $\mathbb{L} \subseteq \mathbb{M}$ induces a Banach lattice homomorphism $\hat{i}: FBL\langle \mathbb{L} \rangle \rightarrow FBL\langle \mathbb{M} \rangle$ between the corresponding free Banach lattices generated by these lattices. We show that this mapping \hat{i} might not be an isomorphic embedding. Sufficient conditions for \hat{i} to be an isometric embedding are provided by considering a notion of locally complemented lattices. As a consequence, we obtain that every free Banach lattice generated by a lattice is Banach lattice isomorphic to an AM-space.

Furthermore, we prove that \hat{i} is an isomorphic embedding if and only if it is injective, which in turn is equivalent to the fact that every lattice homomorphism $x^*: \mathbb{L} \rightarrow [-1, 1]$ extends to a lattice homomorphism $\hat{x}^*: \mathbb{M} \rightarrow [-1, 1]$. Using this characterization we provide an example of lattices $\mathbb{L} \subseteq \mathbb{M}$ for which \hat{i} is an isomorphic not isometric embedding.

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