

COMPACTNESS OF THE TWO-DIMENSIONAL RECTANGULAR HARDY OPERATOR

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Abstract. Criteria in terms of weights v and w are given for the compactness of the two-dimensional Hardy operator I_2 from Lebesgue space $L_v^p(\mathbb{R}_+^2)$ to $L_w^q(\mathbb{R}_+^2)$ for $1 < p \leq q < \infty$. A two-sided estimate is found for the measure of non-compactness of $I_2 : L_v^p(\mathbb{R}_+^2) \rightarrow L_w^q(\mathbb{R}_+^2)$ for the same case of summation parameters p and q . The situation when $q < p$ is also discussed.

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