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# TRIANGULAR CESÀRO SUMMABILITY AND LEBESGUE POINTS OF TWO–DIMENSIONAL FOURIER SERIES

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*Abstract.* We prove that the triangular Cesàro means of two-dimensional functions $f \in L_1(\mathbb{T}^2)$ converge to $f$ at each strong $(1,\omega)$-Lebesgue point. Moreover, if $f \in L_p(\mathbb{T}^2)$ with $1 < p < \infty$, then the Cesàro means converge to $f$ at each $(p,\omega)$-Lebesgue point. This generalizes the well known classical Lebesgue's theorem.

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