

LOG-CONVEXITY OF GENERALIZED KANTOROVICH FUNCTION

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Abstract. We aim to derive some important properties about the generalized Kantorovich constant $K(h, p) := \frac{h^p - h}{(p-1)(h-1)} \left(\frac{p-1}{p} \frac{h^p - 1}{h^p - h} \right)^p$ for $h > 0$ and $p \in \mathbb{R}$. In particular, we point out that $K(h, p)$ is a log-convex function for p . As applications, we show the monotonicity of $K(h, p)^{\frac{1}{p}}$.

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