

## SPACEABILITY ON SOME CLASSES OF BANACH SPACES

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*Abstract.* In this paper, we study spaceability of subsets of generalized Orlicz and Lebesgue spaces associated to a Banach function space. Also, we give some sufficient conditions for spaceability of subsets of a general Banach space which improves an important result on this topic. As an application, it is shown that the set of all bounded linear operators which are not positive semidefinite on a separable Hilbert space is spaceable.

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