

## ON QUASINORMALITY OF THE DILATION OF TRUNCATED TOEPLITZ OPERATORS

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*Abstract.* An operator  $S_{\varphi, \psi}^u$  on  $L^2$  is called the *dilation of a truncated Toeplitz operator* if for two symbols  $\varphi, \psi \in L^\infty$  and an inner function  $u$ ,

$$S_{\varphi, \psi}^u f = \varphi P_u f + \psi Q_u f$$

holds for  $f \in L^2$  where  $P_u$  denotes the orthogonal projection of  $L^2$  onto  $\mathcal{K}_u^2$  and  $Q_u = I - P_u$ . In this paper, we study characterizations for the dilation of truncated Toeplitz operators  $S_{\varphi, \psi}^u$  to be quasinormal. As consequences of the results, we investigate the forms of the symbol functions  $\varphi$  and  $\psi$  when such operator becomes a quasinormal operator.

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### REFERENCES

- [1] J. AGLER AND J. E. MCCARTHY, *Pick interpolation and Hilbert function spaces*, Graduate Studies in Mathematics, Vol. **44**, Amer. Math. Soc., 2002.
- [2] I. CHALENDAR AND D. TIMOTIN, *Commutation relation for truncated Toeplitz operators*, Oper. Matrices, **8** (2014), 877–888.
- [3] J. A. CIMA, W. T. ROSS, AND W. R. WOGEN, *Truncated Toeplitz operators on finite dimensional spaces*, Oper. Matrices, **3** (2) (2008), 357–369.
- [4] S. R. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications*, Trans. Amer. Math. Soc., **358** (2006), 1285–1315.
- [5] C. GU AND D. KANG, *A Commutator Approach to Truncated Singular Integral Operators*, Integr. Equ. Oper. Theory, **90** (16) (2018), 1–22.
- [6] S. R. GARCIA AND W. T. ROSS, *Recent progress on truncated Toeplitz operators*, Blaschke Products and Their Applications, Fields Inst. Commun., **65** (2013), 275–319.
- [7] P. R. HALMOS, *A Hilbert space problem book*, Second edition, Springer-Verlag, New York, 1982.
- [8] E. KO AND J. E. LEE, *On the dilation of truncated Toeplitz operators*, Complex Anal. Op. Th., **10** (2016), 815–817.
- [9] E. KO, J. E. LEE, AND T. NAKAZI, *On the dilation of truncated Toeplitz operators II*, Complex Anal. Op. Th., **13** (2019), 3549–3568.
- [10] E. KO, J. E. LEE, AND T. NAKAZI, *Hyponormality of the dilation of truncated Toeplitz operators*, Complex Var. Elliptic Eq., **66** (10) (2021), 1664–1675.
- [11] S. NAGY, C. FOIAS, H. BERCOVICI, AND L. KERCHY, *Harmonic analysis of operators on Hilbert space*, Springer, 2010.
- [12] D. SARASON, *Algebraic properties of truncated Toeplitz operators*, Oper. Matrices, **1** (2007), 419–526.
- [13] N. A. SEDLOCK, *Algebras of truncated Toeplitz operators*, Oper. Matrices, **5** (2011), 309–326.
- [14] YU. L. SHMUL'YAN, *An operator Hellinger integral*, Mat. Sb. **49** (91) (1959), 381–430 (Russian).