# GRADIENT ESTIMATES FOR THE $p$–LAPLACIAN PARABOLIC EQUATIONS WITH A LOW–ORDER TERM

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*Abstract.* This paper mainly deals with regularity estimates in Orlicz spaces for the following divergence parabolic equations of $p$-Laplacian type with a low-order term

$$u_t - \operatorname{div}\left(|Du|^{p-2}Du\right) + |u|^{p-2}u = \operatorname{div}\left(|\mathbf{f}|^{p-2}\mathbf{f}\right) \quad \text{in } \Omega_T := \Omega \times (0,T)$$

under some proper assumptions on $\mathbf{f}(x,t)$. Remarkably, the equations we have discussed here contain the low-order term $|u|^{p-2}u$.

*Mathematics subject classification* (2020): 35K55, 35K65.
*Keywords and phrases*: $p$-Laplacian, regularity, divergence, Orlicz, parabolic, low-order.

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