

## ON CONVEX AND CONCAVE SEQUENCES AND THEIR APPLICATIONS

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*Abstract.* The aim of this paper is to introduce and to investigate the basic properties of  $q$ -convex,  $q$ -affine and  $q$ -concave sequences and to establish their surprising connection to Chebyshev polynomials of the first and of the second kind. One of the main results shows that  $q$ -concave sequences are the pointwise minima of  $q$ -affine sequences. As an application, we consider a nonlinear selfmap of the  $n$ -dimensional space and prove that it has a unique fixed point. For the proof of this result, we introduce a new norm on the space in terms of a  $q$ -concave sequence and show that the nonlinear operator becomes a contraction with respect to this norm, and hence, the Banach Fixed Point theorem can be applied.

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### REFERENCES

- [1] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge, 1934, (first edition), 1952 (second edition).
- [2] X. Z. KRASNÍQI, *On  $\alpha$ -convex sequences of higher order*, J. Numer. Anal. Approx. Theory **45** (2016), no. 2, 177–182.
- [3] M. KUCZMA, *An Introduction to the Theory of Functional Equations and Inequalities*, Prace Naukowe Uniwersytetu Śląskiego w Katowicach, vol. 489, Państwowe Wydawnictwo Naukowe – Uniwersytet Śląski, Warszawa-Kraków-Katowice, 1985, 2nd edn. (ed. by A. Gilányi), Birkhäuser, Basel, 2009.
- [4] D. S. MITRINOVIĆ, *Analytic inequalities*, Die Grundlehren der mathematischen Wissenschaften, Band 165, Springer-Verlag, New York-Berlin, 1970, In cooperation with P. M. Vasić.
- [5] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Inequalities Involving Functions and Their Integrals and Derivatives*, Mathematics and its Applications (East European Series), vol. 53, Kluwer Academic Publishers Group, Dordrecht, 1991.
- [6] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Classical and New Inequalities in Analysis*, Mathematics and its Applications (East European Series), vol. 61, Kluwer Academic Publishers Group, Dordrecht, 1993.
- [7] C. P. NICULESCU AND L.-E. PERSSON, *Convex Functions and Their Applications*, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, 23, Springer-Verlag, New York, 2006, A contemporary approach.
- [8] M. NIEZGODA, *Remarks on convex functions and separable sequences, II*, Discrete Math. **311** (2011), no. 2–3, 178–185.
- [9] M. NIEZGODA, *Inequalities for convex sequences and nondecreasing convex functions*, Aequationes Math. **91** (2017), no. 1, 1–20.
- [10] M. NIEZGODA, *Sherman, Hermite-Hadamard and Fejér like inequalities for convex sequences and nondecreasing convex functions*, Filomat **31** (2017), no. 8, 2321–2335.
- [11] T. POPOVICIU, *Les fonctions convexes*, Hermann et Cie, Paris, 1944.
- [12] A. W. ROBERTS AND D. E. VARBERG, *Convex Functions*, Pure and Applied Mathematics, vol. 57, Academic Press, New York-London, 1973.
- [13] D. F. SOFONEA, I. ȚINCU, AND A. M. ACU, *Convex sequences of higher order*, Filomat **32** (2018), no. 13, 4655–4663.

- [14] JA. TABOR, JÓ. TABOR, AND M. ŽOLDAK, *Strongly convex sequences*, Inequalities and applications 2010, Internat. Ser. Numer. Math., vol. 161, Birkhäuser/Springer, Basel, 2012, p. 183–188.
- [15] SH. WU AND L. DEBNATH, *Inequalities for convex sequences and their applications*, Comput. Math. Appl. **54** (2007), no. 4, 525–534.
- [16] Ş. YILDIZ, *A general matrix application of convex sequences to Fourier series*, Filomat **32** (2018), no. 7, 2443–2449.