

## AVERAGE SAMPLING AND RECONSTRUCTION IN SHIFT-INVARIANT SUBSPACES OF MIXED LEBESGUE SPACE $L^{p,q}(\mathbb{R}^{d+1})$

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*Abstract.* In this paper, we mainly study the average sampling and reconstruction for signals in a shift-invariant subspace of mixed Lebesgue space  $L^{p,q}(\mathbb{R}^{d+1})$  with the generator belonging to a mixed Wiener amalgam space. First, the sampling stability for two kinds of average sampling functionals are considered. Second, two kinds of iterative approximation projection reconstruction algorithms with exponential convergence are utilized for recovering the corresponding signals. Finally, error estimations are also considered under three different conditions.

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