

LYAPUNOV-TYPE INEQUALITIES FOR A NONLINEAR SEQUENTIAL FRACTIONAL BVP IN THE FRAME OF GENERALIZED HILFER DERIVATIVES

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Abstract. We consider a nonlinear sequential fractional boundary value problem (BVP) in the frame of generalized ψ -Hilfer derivatives. We obtain the Green function and some of its properties, from which we derive a new Lyapunov-type inequality for our problem. As a consequence, we present a lower bound for the eigenvalues of the problem. We give some existence results. We emphasize that our results are still valid for some other classes of source functions having some singularities.

Mathematics subject classification (2020): 26A33, 26D10, 34A08, 34L15.

Keywords and phrases: Fractional derivatives, Lyapunov inequality, existence and uniqueness, eigenvalues.

REFERENCES

- [1] J. BUSTOZ, M. E. H. ISMAIL, *On Gamma Function Inequalities*, Math Comput. 47 (176) (1986), 659–667.
- [2] S. CLARK, D. HINTON, *A Lyapunov inequality for linear Hamiltonian systems*, Math. Inequal. Appl. 1 (1998), 201–209.
- [3] R. A. C. FERREIRA, *A Lyapunov-type inequality for a fractional boundary value problem*, Fract. Calc. Appl. Anal. 16 (4) (2013), 978–984.
- [4] R. A. C. FERREIRA, *On a Lyapunov-type inequality and the zeros of a certain Mittag-Leffler function*, J. Math. Anal. Appl. 412 (2) (2014), 1058–1063.
- [5] R. A. C. FERREIRA, *Novel Lyapunov-type inequalities for sequential fractional boundary value problems*, RACSAM, 113 (1) (2019), 171–179.
- [6] R. A. C. FERREIRA, *Two-point fractional boundary value problems*, J. Integral Equ. Appl. 33 (2), 229–236, Summer 2021, <https://doi.org/10.1216/jie.2021.33.229>.
- [7] A. GRANAS, J. DUGUNDJI, *Fixed point theory*, Springer-Verlag New York, Inc. (2003).
- [8] A. KASSYMOV, T. TOREBEK, *Lyapunov-type inequalities for a nonlinear fractional boundary value problem*, RACSAM, 115, 15 (2021), <https://doi.org/10.1007/s13398-020-00954-9>.
- [9] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and applications of fractional differential equations*, North-Holland mathematics studies, vol. 207 (2006), Amsterdam: Elsevier.
- [10] A. M. LYAPUNOV, *Problème général de la stabilité du mouvement*, Ann. Fac. Sci. Univ. Toulouse 2, (1907), 203–407.
- [11] Q. MA, C. MA, J. WANG, *A Lyapunov-type inequality for a fractional differential equation with Hadamard derivative*, J. Math. Inequal. 11 (2017), 135–141.
- [12] J. P. PINASCO, *Lyapunov-Type Inequalities: With Applications to Eigenvalue Problems*, Springer Briefs in Mathematics, Springer, New York, (2013).
- [13] J. VANTERLER DA C. SOUSA, E. CAPELAS DE OLIVEIRA, *On the ψ -Hilfer fractional derivative*, Commun. Nonlinear Sci. Numer. Simulat. 60 (2018), 72–91.
- [14] X. H. TANG, M. ZHANG, *Lyapunov inequalities and stability for linear Hamiltonian systems*, J. Differential Equations. 252 (2012), 358–381.

- [15] X. YANG, *On Lyapunov-type inequality for certain higher-order differential equations*, Appl. Math. Comput. 134 (2003), 307–317.
- [16] X. YANG, K. LO, *Lyapunov-type inequality for a class of even-order differential equations*, Appl. Math. Comput. 215 (2010), 3884–3890.
- [17] Y. WANG, Q. WANG, *Lyapunov-type inequalities for nonlinear fractional differential equation with Hilfer fractional derivative under multi-point boundary conditions*, Fract. Calc. Appl. Anal. 21 (2018), 833–843.
- [18] W. ZHANG, W. LIU, *Lyapunov-type inequalities for sequential fractional boundary value problems using Hilfer's fractional derivative*, J. Inequal. Appl. 98 (2019), <https://doi.org/10.1186/s13660-019-2050-6>.
- [19] B. F. ZOHRA, H. BENAOUA, K. MOKHTAR, *Lyapunov and Hartman-Wintner-type inequalities for a nonlinear fractional BVP with generalized ψ -Hilfer derivative*, Math. Meth. Appl. Sci. (2020), 1–13, <https://doi.org/10.1002/mma.6590>.