Mathematical Inequalities & Applications
Volume 25, Number 3 (2022), 881–901 doi:10.7153/mia-2022-25-56

ON THE EQUIVALENCE OF STATISTICAL DISTANCES FOR ISOTROPIC CONVEX MEASURES

ARNAUD MARSIGLIETTI AND PUJA PANDEY

Abstract. We establish quantitative comparisons between classical distances for probability distributions belonging to the class of convex probability measures. Distances include total variation distance, Wasserstein distance, Kullback-Leibler distance and more general Rényi divergences. This extends a result of Meckes and Meckes (2014).

Mathematics subject classification (2020): 52A40, 60E15.
Keywords and phrases: Convex measure, total variation distance, Rényi divergence, reverse inequality.

REFERENCES

[1] R. ADAMCZAK, O. GUÉDON, R. LATALA, A. LITVAK, K. OLESZKIEWICZ, A. PAJOR, N. TOMCZAK-JAEGERMANN, *Moment estimates for convex measures*, Electron. J. Probab. 17 (2012), no. 101, 19 pp.

[2] S. ARTSTEIN, K. M. BALL, F. BARTHE, A. NAOR, *On the rate of convergence in the entropic central limit theorem*, Probab. Theory Related Fields 129 (3): 381–390, 2004.

[3] M. F. BALCAN, H. ZHANG, *Sample and Computationally Efficient Learning Algorithms under S-Concave Distributions*, Preprint, 2017, arXiv:1703.07758.

[4] K. BALL, *Volume ratios and a reverse isoperimetric inequality*, J. London Math. Soc. (2), 44 (2): 351–359, 1991.

[5] A. R. BARRON, *Entropy and the central limit theorem*, Ann. Probab. 14 (1): 336–342, 1986.

[6] F. BARTHE, *On a reverse form of the Brascamp-Lieb inequality*, Invent. Math. 134 (1998), 335–361.

[7] A. BECK, *First-order methods in optimization*, MOS-SIAM Series on Optimization, 25. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA; Mathematical Optimization Society, Philadelphia, PA, 2017. xii+475 pp.

[8] L. BERWALD, *Verallgemeinerung eines Mittelwertsatzes von J. Favard für positive konkave Funktionen*, (German), Acta Math. 79, (1947), 17–37.

[9] S. G. BOBKOV, *Large deviations and isoperimetry over convex probability measures with heavy tails*, Electr. J. Probab. 12 (2007), pp. 1072–1100.

[10] S. G. BOBKOV, G. P. CHISTYAKOV, F. GÖTZE, *Rényi divergence and the central limit theorem*, Ann. Probab. 47 (2019), no. 1, pp. 270–323.

[11] S. G. BOBKOV, M. MADIMAN, *The entropy per coordinate of a random vector is highly constrained under convexity conditions*, IEEE Transactions on Information Theory, vol. 57 (2011), no. 8, pp. 4940–4954.

[12] S. G. BOBKOV, M. MADIMAN, *Reverse Brunn-Minkowski and reverse entropy power inequalities for convex measures*, J. Funct. Anal. 262 (2012) no. 7, 3309–3339.

[13] S. G. BOBKOV, A. MARSIGLIETTI, *Entropic CLT for smoothed convolutions and associated entropy bounds*, International Mathematics Research Notices, vol. 2020, no. 21, 8057–8080, 2020.

[14] C. BORELL, *Complements of Lyapunov's inequality*, Math. Ann. 205 (1973), 323–331.

[15] C. BORELL, *Convex measures on locally convex spaces*, Ark. Mat., 12: 239–252, 1974.

[16] C. BORELL, *Convex set functions in d-space*, Period. Math. Hungarica 6 (1975), 111–136.

[17] K. BÖRÖCZKY, D. HUG, *Isotropic measures and stronger forms of the reverse isoperimetric inequality*, Trans. Amer. Math. Soc. 369 (2017), 6987–7019.

Paper MIA-25-56

[18] J. BOURGAIN, *On high-dimensional maximal functions associated to convex bodies*, Amer. J. Math. 108 (1986), no. 6, 1467–1476.

[19] H. BRÉZIS, *Propriétés régularisantes de certains semi-groupes non linéaires*, Israel J. Math. 9 (1971) 513–534.

[20] I. CSISZÀR, *Information-type measures of difference of probability distributions and indirect observations*, Studia Sci. Math. Hungar. 2 (1967), 299–318.

[21] P. CATTIAUX, A. GUILLIN, *On the Poincaré constant of log-concave measures*, Geometric Aspects of Functional Analysis: Israel Seminar (GAFA), 2017–2019, vol. I, LNM 2256, Springer Verlag, 171–217, 2020.

[22] T. VAN ERVEN, P. HARREMOËS, *Rényi divergence and Kullback-Leibler divergence*, IEEE Trans. Inform. Theory 60 (2014), no. 7, 3797–3820.

[23] R. ELDAN, B. KLARTAG, *Pointwise Estimates for Marginals of Convex Bodies*, J. Functional Analysis, vol. 254, issue 8, (2008), 2275–2293.

[24] R. ELDAN, D. MIKULINCER, A. ZHAI, *The CLT in high dimensions: quantitative bounds via martingale embedding*, preprint, arXiv:1806.09087, 2018.

[25] J. FAVARD, *Sur les valeurs moyennes*, Bull. Sci. Math. (2) 57 (1933), 54–64.

[26] M. FRADELIZI, *Contributions à la géométrie des convexes – Méthodes fonctionnelles et probabilistes*, preprint, 2008, available at: https://perso.math.u-pem.fr/fradelizi.matthieu/pdf/HDR.pdf.

[27] M. FRADELIZI, J. LI, M. MADIMAN, *Concentration of information content for convex measures*, Electronic Journal of Probability, 25 (2020) paper no. 20, 22 pp.

[28] M. FRADELIZI, O. GUÉDON, *A generalized localization theorem and geometric inequalities for convex bodies*, Advances in Mathematics 204 (2006), 509–529.

[29] P. FRANK, G. PICK, *Distanzschätzungen im Funktionenraum I*, Math. Ann. 76 (1915), no. 2–3, 354–375.

[30] G. L. GILARDONI, *On Pinsker's and Vajda's type inequalities for Csiszar's f-divergences*, IEEE Trans. Inform. Theory 56 (2010), no. 11, 5377–5386.

[31] B. GRÜNBAUM, *Partitions of mass-distributions and of convex bodies by hyperplanes*, Pacific J. Math. 10 (1960), 1257–1261.

[32] D. HENSLEY, *Slicing convex bodies – bounds for slice area in terms of the body's covariance*, Proc. Amer. Math. Soc. 79 (1980), no. 4, 619–625.

[33] A. JOURANI, L. THIBAULT, D. ZAGRODNY, *Differential properties of the Moreau envelope*, (English summary), J. Funct. Anal. 266 (2014), no. 3, 1185–1237.

[34] S. KARLIN, F. PROSCHAN, R. E. BARLOW, *Moment inequalities of Pólya frequency functions*, Pacific J. Math. 11 (1961), 1023–1033.

[35] B. KLARTAG, *A central limit theorem for convex sets*, Invent. Math., vol. 168, (2007), 91–131.

[36] M. LEDOUX, *Spectral gap, logarithmic Sobolev constant, and geometric bounds, in: Surveys Differ*, Geom., vol. IX, Int. Press, Somerville, MA, 2004, pp. 219–240.

[37] L. LEINDLER, *On a certain converse of Hölder's inequality, II*, Acta Sci. Math., 33 (1972), 217–223.

[38] K. MAHLER, *Ein Übertragungsprinzip für konvexe Körper*, Casopis Pest. Mat. Fys. 68 (1939), 93–102.

[39] A. MARSIGLIETTI, V. KOSTINA, *New connections between the entropy power inequality and geometric inequalities*, Proceedings 2018 IEEE International Symposium on Information Theory, Vail, Colorado, June 2018.

[40] E. MECKES, M. MECKES, *On the Equivalence of Modes of Convergence for Log-Concave Measures*, In Geometric aspects of functional analysis (2011/2013), vol. 2116 of Lecture Notes in Math., pages 385–394, Springer, Berlin, 2014.

[41] D. MILMAN, *Inégalité de Brunn-Minkowski inverse et applications à la théorie locale des espaces normés*, C. R. Acad. Sci. Paris Sér. I Math., 302 (1): 25–28, 1986.

[42] V. D. MILMAN, *Isomorphic symmetrizations and geometric inequalities*, In Geometric aspects of functional analysis (1986/87), vol. 1317 of Lecture Notes in Math., pages 107–131, Springer, Berlin, 1988.

[43] J. J. MOREAU, *Proximité et dualité dans un espace hilbertien*, Bull. Soc. Math. France 93 (1965) 273–299.

[44] M. S. PINSKER, *Information and information stability of random variables and processes*, Translated and edited by Amiel Feinstein Holden-Day, Inc., San Francisco, Calif.-London-Amsterdam, 1964, xii+243 pp.
[45] G. PISIER, *The volume of convex bodies and Banach space geometry*, vol. 94 of Cambridge Tracts in Mathematics, Cambridge University Press, Cambridge, 1989.
[46] M. TALAGRAND, *Transportation cost for Gaussian and other product measures*, Geom. Funct. Anal. 6 (1996), no. 3, 587–600.
[47] C. VILLANI, *Optimal Transport: Old and New*, vol. 338 of Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], Springer-Verlag, Berlin, 2009.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com