

VOLTERRA INTEGRAL OPERATOR FROM WEIGHTED BERGMAN SPACES TO GENERAL FUNCTION SPACES

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Abstract. The boundedness, compactness and essential norm of Volterra integral operator V_g from weighted Bergman spaces A_{α}^p to general function spaces $F(q, qt - 2, s)$ are investigated in this paper.

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