

HARDY–HILBERT TYPE INEQUALITIES FOR MATRICES

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Abstract. By making use of the weighted geometric mean of positive definite matrices, we extend Hardy–Hilbert type inequalities to matrix case. Our results complement those of Hansen [Internat. J. Math. 21 (2010), no. 10, 1283–1295] and Kian [Ann. Funct. Anal. 3 (2012), no. 2, 128–134].

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