

TAYLOR-TYPE EXPANSIONS IN TERMS OF EXPONENTIAL POLYNOMIALS

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Abstract. The aim of this paper is to derive an extension of the Taylor theorem related to linear differential operators with constant coefficients. For this aim, using divided differences with repeated arguments, the so-called characteristic element from the kernel of the differential operator is described. The extension of the Taylor theorem related to exponential polynomials and its consequences are established with integral remainder terms as well as in the form of mean value type theorems.

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