

HARDY–LITTLEWOOD–STEIN INEQUALITIES FOR DOUBLE TRIGONOMETRIC SERIES

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Abstract. In the present paper, we obtain sharper analogues of the Hardy-Littlewood-Stein inequalities for double trigonometric series. We also establish a new unified version of the Hardy-Littlewood-Stein inequalities for Fourier series in regular systems, which covers the whole range $1 < p < \infty$ including the critical case $p = 2$.

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