Mathematical
Inequalities
& Applications
Volume 26, Number 1 (2023), 69–82 doi:10.7153/mia-2023-26-06

A SUFFICIENT CONDITION FOR A COMPLEX POLYNOMIAL TO HAVE ONLY SIMPLE ZEROS AND AN ANALOG OF HUTCHINSON'S THEOREM FOR REAL POLYNOMIALS

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Abstract. We find the constant b_∞ ($b_\infty \approx 4.81058280$) such that if a complex polynomial or entire function $f(z) = \sum_{k=0}^{\omega} a_k z^k$, $\omega \in \{2,3,4,\ldots\} \cup \{\infty\}$, with nonzero coefficients satisfy the conditions $\left|\frac{a_k^2}{a_{k-1}a_{k+1}}\right| > b_\infty$ for all $k = 1,2,\ldots,\omega-1$, then all the zeros of f are simple. We show that the constant b_∞ in the statement above is the smallest possible. We also obtain an analog of Hutchinson's theorem for polynomials or entire functions with real nonzero coefficients.

Mathematics subject classification (2020): 30C15, 26C10, 30D15.
Keywords and phrases: Complex polynomial, entire function, simple zeros, Hutchinson's theorem, second quotients of Taylor coefficients.

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