

## THE GENERALIZED GAO'S CONSTANT OF ABSOLUTE NORMALIZED NORMS IN $\mathbb{R}^2$

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*Abstract.* In this paper, we give the method to calculate the generalized Gao's constant under the absolute normalized norms in  $\mathbb{R}^2$ . Using this method, we can compute the exact values of the generalized Gao's constant of some concrete Banach spaces easily, such as Banach lattice, Lorentz sequence spaces etc.

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