

UNIFORM POINCARÉ INEQUALITY IN O-MINIMAL STRUCTURES

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Abstract. We first define the trace on a domain Ω which is definable in an o-minimal structure. We then show that every function $u \in W^{1,p}(\Omega)$ vanishing on the boundary in the trace sense satisfies Poincaré inequality. We finally show, given a definable family of domains $(\Omega_t)_{t \in \mathbb{R}^k}$, that the constant of this inequality remains bounded, if so does the volume of Ω_t .

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