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ON THE IRREGULARITY OF GRAPHS BASED ON THE ARITHMETIC–GEOMETRIC MEAN INEQUALITY

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Abstract. For a graph G of order n, size m and degree sequence $\mathscr{D}(G) = (d_1, d_2, \ldots, d_n)$, a new measure of irregularity

$$\mathrm{I}_{AG}(G) = 1 - n^n (d_1 + r)(d_2 + r) \cdots (d_n + r)/(2m + rn)^n,$$

$r \in \mathbb{R}_{\geqslant 0}$, is introduced. It is shown that if G has maximum I_{AG}-irregularity among all connected graphs of order n and size m, then (i) $\Delta(G) = n-1$; (ii) for each $u, v \in V(G)$ with the property $d_G(u) \leqslant d_G(v)$, it holds that $N(G,u) \subseteq N[G,v]$, where $N(G,w)$ and $N[G,w]$ are the neighbourhood and the closed neighbourhood of w in G, respectively; (iii) G is a threshold graph. Further, it is proven that if a graph H has a minimum value of I_{AG}-irregularity among all irregular graphs of the same order and size, then $\Delta(H) - \delta(H) = 1$. Finally, the graphs with minimum and maximum I_{AG}-irregularity in the classes of trees, unicyclic and bicyclic graphs are characterized.

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