

ESTIMATING THE HARDY CONSTANT OF NONCONCAVE GINI MEANS

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Abstract. The extension of the Hardy-Knopp-Carleman inequality to several classes of means was the subject of numerous papers. In the class of Gini means the Hardy property was characterized in 2015 by the second author. The precise value of the associated Hardy constant was only established for concave Gini means by the authors in 2016. The determination of the Hardy constant for nonconcave Gini means is still an open problem. The main goal of this paper is to establish sharper upper bounds for the Hardy constant in this case. The method is to construct a homogeneous and concave quasideviation mean which majorizes the nonconcave Gini mean and for which the Hardy constant can be computed.

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