

DIRAC INEQUALITY FOR HIGHEST WEIGHT HARISH-CHANDRA MODULES I

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Abstract. Let G be a connected simply connected noncompact classical simple Lie group of Hermitian type. Then G has unitary highest weight representations. The proof of the classification of unitary highest weight representations of G given by Enright, Howe and Wallach is based on the Dirac inequality of Parthasarathy, Jantzen's formula and Howe's theory of dual pairs where one group in the pair is compact. In this paper we focus on the Dirac inequality which can be used to prove the classification in a more direct way.

Mathematics subject classification (2020): 22E47.

Keywords and phrases: Unitary highest weight modules, Dirac inequality.

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