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OPTIMAL DIVISIONS OF A CONVEX BODY

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Abstract. For a convex body C in $\mathbb{R}^d$ and a division of C into convex subsets $C_1,\dots,C_n$, we can consider $max\{F(C_1),\dots,F(C_n)\}$ (respectively, $min\{F(C_1),\dots,F(C_n)\}$), where F represents one of these classical geometric magnitudes: the diameter, the minimal width, or the inradius. In this work we study the divisions of C minimizing (respectively, maximizing) the previous value, as well as other related questions.

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