TURÁN'S INEQUALITY FOR
ULTRASPHERICAL POLYNOMIALS REVISITED

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Abstract. We present a short proof that the normalized Turán determinant in the ultraspherical case is convex or concave depending on whether parameter λ is positive or negative.

Mathematics subject classification (2020): Primary 33C45; Secondary 42C05.

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