

SHERMAN'S FUNCTIONAL, ITS PROPERTIES WITH APPLICATIONS FOR f -DIVERGENCE MEASURE

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Abstract. In this paper we define Sherman's functional deduced from Sherman's inequality. We established lower and upper bounds for Sherman's functional and study its properties. As consequences of main results we obtained new bounds for Csiszár f -divergence functional and as special cases bounds for Shannon's entropy. As applications we use the Zipf-Mandelbrot law to introduce a new entropy and to derive some related results.

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