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INEQUALITIES IN TIME–FREQUENCY ANALYSIS

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Abstract. Different types of Nash inequaliy, Sobolev inequality, Pitt inequality, logarithmic Sobolev inequality and Gross inequality are proved for the short time Fourier transform. Also, several formulations of Beckner's logarithmic uncertainty principle are established for the same transform.

Mathematics subject classification (2020): 42A38, 42B10.

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