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EQUIVALENT QUASI–NORMS ON GENERALIZED ORLICZ SPACES

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Abstract. In this paper we show that the equivalence among the classical quasi-norms of the generalized Orlicz spaces X^{Φ} — the Orlicz quasi-norm, the Luxemburg quasi-norm and the Amemiya quasi-norm — holds under some mild conditions on the underlying quasi-Banach function space X — mainly the weak Fatou property — improving previous results of [4] for which some lattice convexity requirements for the quasi-Banach function space X were needed.

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