CHARACTERIZATIONS OF LIPSCHITZ FUNCTIONS
VIA THE COMMUTATORS OF MAXIMAL FUNCTION
IN ORLICZ SPACES ON STRATIFIED LIE GROUPS

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Abstract. We give necessary and sufficient conditions for the boundedness of the maximal commutators M_b, the commutators of the maximal operator $[b,M]$ and the commutators of the sharp maximal operator $[b,M^\sharp]$ in Orlicz spaces $L^\Phi(\mathbb{G})$ on any stratified Lie group $\mathbb{G}$ when b belongs to Lipschitz spaces $\dot{\Lambda}_\beta(\mathbb{G})$. We obtain some new characterizations for certain subclasses of Lipschitz spaces $\dot{\Lambda}_\beta(\mathbb{G})$.

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