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ON THE CONSTANT IN THE HARDY INEQUALITY FOR FINITE SEQUENCES

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Abstract. We investigate the behaviour of the smallest possible constant d_n in the Hardy's inequality

$$\sum_{k=1}^{n}\Big(\frac{1}{k}\sum_{j=1}^{k}a_j\Big)^2 \leqslant d_n\sum_{k=1}^{n}a_k^2, \qquad (a_1,\dots,a_n)\in\mathbb{R}^n.$$

A new proof of the Hardy's inequality is given which allows us to give another much simpler proof of the upper estimation of d_n

$$d_n < 4-\frac{c}{\ln^2 n}, \qquad c>0.$$

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