WEIGHTED DYNAMIC ESTIMATES FOR CONVEX
AND SUBHARMONIC FUNCTIONS ON TIME SCALES

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Abstract. This article introduces a new type of weighted square delta integral inequalities involving the delta derivative of a convex function. As an extension, we also establish weighted square delta integral inequalities for subharmonic functions on time scales. Here, we rely on a new definition of the time scales Laplace operator. The significance of this work in the existing literature is provided at the end of the article.

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