

## INEQUALITIES FOR WEIGHTED SPACES WITH VARIABLE EXPONENTS

PABLO ROCHA

*Abstract.* In this article we obtain an “off-diagonal” version of the Fefferman-Stein vector-valued maximal inequality on weighted Lebesgue spaces with variable exponents. As an application of this result and the atomic decomposition developed in [16] we prove, for certain exponents  $q(\cdot)$  in  $\mathcal{S}^{\log}(\mathbb{R}^n)$  and certain weights  $\omega$ , that the Riesz potential  $I_\alpha$ , with  $0 < \alpha < n$ , can be extended to a bounded operator from  $H_\omega^{p(\cdot)}(\mathbb{R}^n)$  into  $L_\omega^{q(\cdot)}(\mathbb{R}^n)$ , for  $\frac{1}{p(\cdot)} := \frac{1}{q(\cdot)} + \frac{\alpha}{n}$ .

*Mathematics subject classification (2020):* 42B30, 42B25, 42B35, 46E60.

*Keywords and phrases:* Fefferman-Stein inequalities, weighted variable Hardy spaces, atomic decomposition, Riesz potential.

### REFERENCES

- [1] H. Q. BUI, *Weighted Hardy spaces*, Math. Nachr. **103** (1981), 45–62.
- [2] C. CAPONE, D. CRUZ-URIBE AND A. FIORENZA, *The fractional maximal operator and fractional integral on variable  $L^p$  spaces*, Rev. Mat. Iberoam. **23** (3) (2007), 743–770.
- [3] R. COIFMAN, *A real characterization of  $H^p$* , Studia Math. **51** (1974), 269–274.
- [4] R. COIFMAN AND G. WEISS, *Extensions of Hardy spaces and their use in analysis*, Bull. Amer. Math. Soc. **83** (1977), 569–645.
- [5] D. CRUZ-URIBE, A. FIORENZA AND C. NEUGEBAUER, *Weighted norm inequalities for the maximal operator on variable Lebesgue spaces*, J. Math. Anal. Appl. **394** (2012), 744–760.
- [6] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue Spaces*, Birkhäuser, 2013.
- [7] D. CRUZ-URIBE AND D. WANG, *Variable Hardy Spaces*, Indiana Univ. Math. J. **63** (2) (2014), 447–493.
- [8] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RUŽIČKA, *Lebesgue and Sobolev spaces with Variable Exponents*, Springer, 2011.
- [9] L. DIENING AND P. HÄSTÖ, *Muckenhoupt weights in variable exponent spaces*, Preprint (2010).
- [10] Y. DING, M.-Y. LEE AND C.-C. LIN, *Fractional integrals on weighted Hardy spaces*, J. Math. Anal. Appl. **282** (1) (2003), 356–368.
- [11] C. FEFFERMAN AND E. STEIN,  *$H^p$  spaces of several variables*, Acta Math. **129** (1972), 137–193.
- [12] J. GARCÍA-CUERVA, *Weighted  $H^p$  spaces*, Dissertations Math. **162** (1979), 1–63.
- [13] L. GRAFAKOS, *Classical Fourier Analysis*, 3rd edition, Graduate Texts in Mathematics, 249, Springer New York, 2014.
- [14] E. HARBOURE, R. MACÍAS AND C. SEGOVIA, *Extrapolation results for classes of weights*, Amer. J. Math. **110** (3) (1988), 383–397.
- [15] K.-P. HO, *Singular integral operators, John-Nirenberg inequalities and Triebel-Lizorkin type spaces on weighted Lebesgue spaces with variable exponents*, Rev. Un. Mat. Argentina **57** (1) (2016), 85–101.
- [16] K.-P. HO, *Atomic decompositions of weighted Hardy spaces with variable exponents*, Tohoku Math. J. **(2) 69** (3) (2017), 383–413.
- [17] K.-P. HO, *Sublinear operators on weighted Hardy spaces with variable exponents*, Forum Math. **31** (3) (2019), 607–617.
- [18] K.-P. HO, *Fractional integral operators on Orlicz slice Hardy spaces*, Fract Calc Appl Anal **25** (2022), 1294–1305.
- [19] O. KOVÁČIK AND J. RÁKOSNÍK, *On spaces  $L^{p(x)}$  and  $W^{k,p(x)}$* , Czechoslovak Math. J. **41** (116) (1991), 592–618.

- [20] S. KRANTZ, *Fractional integration on Hardy spaces*, *Studia Mathematica*, vol **73** (2) (1982), 87–94.
- [21] R. H. LATTER, *A characterization of  $H^p(\mathbb{R}^n)$  in terms of atoms*, *Studia Math.* **62** (1978), 93–101.
- [22] M.-Y. LEE AND C.-C. LIN, *The molecular characterization of weighted Hardy spaces*, *Journal of Funct. Analysis*, **188** (2002), 442–460.
- [23] X. LI AND L. PENG, *The molecular characterization of weighted Hardy spaces*, *Science in China (Series A)*, vol. **44** (2) (2001), 201–211.
- [24] B. MUCKENHOUPT AND R. WHEEDEN, *Weighted norm inequalities for fractional integrals*, *Trans. Amer. Math. Soc.*, **192** (1974), 261–274.
- [25] E. NAKAI AND Y. SAWANO, *Hardy spaces with variable exponents and generalized Campanato spaces*, *J. Funct. Anal.* **262** (2012), 3665–3748.
- [26] W. ORLICZ, *Über konjugierte Exponentenfolgen*, *Studia Math.* **3** (1931), 200–211.
- [27] P. ROCHA, *A note on Hardy spaces and bounded linear operators*, *Georgian Math. J.* **25** (1) (2018), 73–76.
- [28] P. ROCHA, *Boundedness of generalized Riesz potentials on the variable Hardy spaces*, *J. Aust. Math. Soc.* **104** (2018), 255–273.
- [29] P. ROCHA, *On the atomic and molecular decomposition of weighted Hardy spaces*, *Rev. Un. Mat. Arg.*, vol. **61** (2) (2020), 229–247.
- [30] P. ROCHA AND M. URCIUOLO, *Fractional type integral operators on variable Hardy spaces*, *Acta Math. Hung.* **143** (2) (2014), 502–514.
- [31] E. STEIN, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton University Press, 1993.
- [32] E. STEIN AND G. WEISS, *On the theory of harmonic functions of several variables I: The theory of  $H^p$  spaces*, *Acta Math.* **103** (1960), 25–62.
- [33] M. H. TAIBLESON AND G. WEISS, *The molecular characterization of certain Hardy spaces*, *Astérisque* **77** (1980), 67–149.
- [34] J.-O. STRÖMBERG AND A. TORCHINSKY, *Weighted Hardy Spaces*, *Lecture Notes in Mathematics*, 1381, Springer-Verlag, Berlin, 1989.
- [35] Y. ZHANG, D. YANG, W. YUAN AND S. WANG, *Real-variable characterizations of Orlicz-slice Hardy spaces*, *Anal. Appl.* **17** (2019), 597–664.