REGULARITY IN ORLICZ SPACES FOR QUASI–LINEAR ELLIPTIC EQUATIONS OF SCHRÖDINGER TYPE

NGUYEN NGOC TRONG, NGUYEN THANH TUNG,
TRAN TRI DUNG AND LE XUAN TRUONG*

Abstract. In this paper, we generalize gradient estimates in Lebesgue spaces to Orlicz spaces for weak solutions of quasi-linear elliptic equations of Schrödinger type on a Reifenberg flat domain, under the condition that the coefficients are in John-Nirenberg space with small *BMO* semi-norms. We assume that the potential belongs to some certain reverse Hölder class. Our results improve the known results for such equations using a harmonic analysis-free technique.

REFERENCES

[1] R. A. ADAMS, *Sobolev spaces*, Pure and Applied Mathematics, Vol. 65. Academic Press, New York-London, 1975.

[2] N. BADR AND B. B. ALI, *Lp boundedness of the Riesz transform related to Schrodinger operators on a manifold*, Ann. Sc. Norm. Super. Pisa Cl. Sci. **5**, 8, 2009.

[3] S. BYUN AND L. WANG, *Elliptic equations with BMO coefficients in Reifenberg domains*, Comm. Pure Appl. Math. **57**, 10 (2004), 1283–1310.

[4] S. BYUN AND L. WANG, *Lp estimates for parabolic equations in Reifenberg domains*, J. Funct. Anal. **223**, 1 (2005) 44–85.

[5] S. BYUN AND L. WANG, *Parabolic equations in time dependent Reifenberg domains*, Adv. Math. **212**, 2 (2007), 797–818.

[6] S. BYUN, L. WANG AND S. ZHOU, *Nonlinear elliptic equations with small BMO coefficients in Reifenberg domains*, J. Funct. Anal. **250**, 1 (2007), 167–196.

[7] S. BYUN, F. YAO AND S. ZHOU, *Gradient estimates in Orlicz space for nonlinear elliptic equations*, J. Funct. Anal. **255**, 8 (2008), 1851–1873.

[8] G. DAVID, C. KENIG AND T. TORO, *Asymptotically optimally doubling measures and Reifenberg flat sets with vanishing constant*, Comm. Pure. Appl. Math. **54**, 4 (2001), 385–449.

[9] F. W. GEHRING, *The L^p-integrability of the partial derivatives of a quasiconformal mapping*, Acta Math **130**, (1973), 265–277.

[10] M. GIAQUINTA, *Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems*, Princeton University Press (AM-105), New Jersey, 1983.

[11] P. W. JONES, *Quasiconformal mappings and extendability of functions in Sobolev spaces*, Acta Math **147**, (1981), 71–88.

[12] C. KENIG AND T. TORO, *Free boundary regularity for harmonic measures and the Poisson kernel*, Ann. of Math. **150**, 2 (1999), 369–454.

[13] T. KILPELÄINEN AND P. KOSKELA, *Global integrability of the gradients of solutions to partial differential equations*, Nonlinear Anal. Theory Methods Appl. **23**, 7 (1994), 899–909.

[14] J. KINNUNEN AND S. ZHOU, *A local estimate for nonlinear equations with discontinuous coefficients*, Comm. Partial Differ. Equ. **24**, 11–12 (1999), 2043–2068.

[15] J. KINNUNEN AND S. ZHOU, *A boundary estimate for nonlinear equations with discontinuous coefficients*, Differ. Integral Equ. **14**, 4 (2001), 475–492.

[16] N. V. KRYLOV, *Parabolic equations with VMO coefficients in Sobolev spaces with mixed norms*, J. Funct. Anal. **250**, 2 (2007), 521–558.

[17] M. LEE AND J. OK, *Interior and boundary $W^{1,q}$-estimates for quasi-linear elliptic equations of Schrödinger type*, J. Differ. Equ. **269**, 5 (2020), 4406–4439.

[18] B. MUCKENHOUPT, *Weighted norm inequalities for the Hardy maximal function*, Trans. Am. Math. Soc. **165** (1972), 207–226.

[19] J. MUSIELAK, *Orlicz spaces and Modular spaces*, Lecture Notes Math 1034, Springer-Verlag, New York/Berlin, (1983), 1–222.

[20] M. RAO AND Z. REN, *Applications of Orlicz Spaces*, CRC Press, Boca Raton, 2002.

[21] E. REINFENBERG, *Solutions of the plateau problem for m-dimensional surfaces of varying topological type*, Acta Math. **104**, (1960), 1–92.

[22] T. TORO, *Doubling and flatness: Geometry of measures*, Notices Amer. Math. Soc. **44**, (1997), 1087–1094.

[23] N. M. VEMPATI, *Weighted Inequalities on Spaces of Homogeneous Type*, Arts & Sciences Electronic Theses and Dissertations **2467**, (2021).

[24] F. YAO, *Optimal regularity for Schrödinger equations*, Nonlinear Anal. **71**, (2009), 5144–5150.

[25] F. YAO, *Regularity theory for the uniformly elliptic operators in Orlicz spaces*, Comput. Math. Appl. **60**, (2010), 3098–3104.

[26] K. ZHANG, *Regularity in Orlicz spaces for nondivergence elliptic operators with potentials satisfying a reverse Holder condition*, Electron. J. Qual. Theory Differ. Equ. **78**, (2013), 1–16.

[27] S. ZHENG, X. ZHENG AND Z. FENG, *Regularity for a class of degenerate elliptic equations with discontinuous coefficients under natural growth*, J. Math. Anal. Appl. **346**, 2 (2008), 359–373.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com