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WEIGHTED SHIFTS ON DIRECTED TREES WITH ONE BRANCHING VERTEX: BETWEEN QUASINORMALITY AND PARANORMALITY

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Abstract. Let $\mathscr{T}_{\eta,\kappa}$ be a directed tree consisting of one branching vertex, η branches and a trunk of length κ and let $S_{\boldsymbol{\lambda}}$ be the associated weighted shift on $\mathscr{T}_{\eta,\kappa}$ with positive weight sequence $\boldsymbol{\lambda}$. Introduced recently was a collection of classical weighted shifts, "the i-th branching weighted shifts" $W^{(i)}$ for $0 \leqslant i \leqslant \eta$, whose weights are derived from those of $S_{\boldsymbol{\lambda}}$ by slicing the branches of the tree $\mathscr{T}_{\eta,\kappa}$ ([9]). As a contrast contrasting to "slicing" we consider "collapsing the branches of a tree" and define "the k-step collapsed weighted shift" $S_{\widetilde{\boldsymbol{\lambda}}^{(k)}}$ on $\mathscr{T}_{\eta-k,\kappa}$ for $1 \leqslant k \leqslant \eta-1$ so that $S_{\widetilde{\boldsymbol{\lambda}}^{(\eta-1)}}$ may become the basic branching shift $W^{(0)}$. In this paper we discuss the relationships between operator properties of $S_{\boldsymbol{\lambda}}$ such as quasinormality, subnormality, ∞-hyponormality, p-hyponormality, and p-paranormality, and these properties for the $W^{(i)}$ and $S_{\widetilde{\boldsymbol{\lambda}}^{(k)}}$.

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