A NOTE ON THE CONVOLUTION IN ORLICZ SPACES

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Abstract. Let G be a locally compact group. In this paper, for given concave Orlicz functions Φ and Ψ with $\limsup_{t\to\infty}\Phi(t)/t = 0$, we prove that the convolution $f * g$ exists, for $f \in L^{\Phi}(G)$ and $g \in L^{\Psi}(G)$, if and only if G is discrete. This extends and completes some recent results concerning the determination of when an Orlicz space on a locally compact group is closed under convolution multiplication.

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