

THE ε -MAXIMAL OPERATOR AND HAAR MULTIPLIERS ON VARIABLE LEBESGUE SPACES

DAVID CRUZ-URIBE, OFS* AND MICHAEL PENROD

Abstract. Recently, Stockdale, Villarroya, and Wick introduced the ε -maximal operator to prove the Haar multiplier is bounded on the weighted spaces $L^p(w)$ for a class of weights larger than A_p . We prove the ε -maximal operator and Haar multiplier are bounded on variable Lebesgue spaces $L^{p(\cdot)}(\mathbb{R}^n)$ for a larger collection of exponent functions than the log-Hölder continuous functions used to prove the boundedness of the maximal operator on $L^{p(\cdot)}(\mathbb{R}^n)$. We also prove that the Haar multiplier is compact when restricted to a dyadic cube Q_0 .

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