

BOUNDEDNESS OF INTEGRAL OPERATORS OF DOUBLE PHASE

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Abstract. Our aim in this note is to establish a Sobolev-type inequality and Trudinger-type inequality for fractional maximal and Riesz potential operators in the framework of general double phase functionals given by

$$\varphi(x, t) = \varphi_1(t) + \varphi_2(b(x)t), \quad x \in \mathbb{R}^n, \quad t \geq 0,$$

where φ_1, φ_2 are positive convex functions on $(0, \infty)$ and b is a nonnegative function on $[0, \infty)$ which is Hölder continuous of order $\theta \in (0, 1]$.

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