

CONSTRUCTING RIESZ–FISCHER SEQUENCES FROM A MINIMAL SEQUENCE IN A HILBERT SPACE \mathcal{H}

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Abstract. In this paper we prove that if $U = \{u_n\}$ is a minimal sequence in a separable Hilbert space \mathcal{H} , then multiplying each vector u_n by an appropriate constant c_n , yields a family of functions $P := \{c_n \cdot u_n\}$, such that P is a Riesz-Fischer sequence in \mathcal{H} .

If U is a minimal and complete sequence in \mathcal{H} , therefore having a unique biorthogonal sequence $V = \{v_n\}$ in \mathcal{H} , then P is a complete Riesz-Fischer sequence in \mathcal{H} and its unique biorthogonal family $\{v_n/\bar{c}_n\}$ is a Bessel sequence in \mathcal{H} .

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