THE POPOVICIU TYPE INEQUALITIES FOR
s–CONVEX FUNCTIONS IN THE THIRD SENSE

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Abstract. In this article, firstly, some examples involving hypergeometric functions for the s-convex functions in the third sense are presented. Then, the Popoviciu type inequalities and some integral versions of these inequalities for the s-convex functions in the third sense are proved. In the last section, by using these obtained inequalities and given sample functions, inequality relations for special functions including beta, incomplete beta, hypergeometric, exponential integral, logarithmic integral functions and for special means including Stolarsky, power, arithmetic, Heronian, geometric means are derived.

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