

ESTIMATIONS OF COVERING FUNCTIONALS OF SIMPLICES

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Abstract. Let S_n be an n -dimensional simplex and $\Gamma_p(S_n)$ be the smallest positive number γ such that S_n can be covered by p translates of γS_n . We obtain an upper bound of the least positive number β such that $-S_n$ can be covered by two translates of βS_n , which is tight when $n = 3$. In addition, we get the exact value of $\Gamma_{n+2}(S_n)$ and an upper bound of $\Gamma_{n+3}(S_n)$. We also provide the precise value of $\Gamma_6(S_3)$, new lower and upper bounds of $\Gamma_7(S_3)$, and an upper bound of $\Gamma_8(S_3)$.

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