

DISCRETE OPIAL TYPE INEQUALITIES FOR INTERVAL-VALUED FUNCTIONS

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Abstract. We introduce the forward (backward) gH-difference operator of interval sequences, and establish some new discrete Opial type inequalities for interval-valued functions. Further, we obtain generalizations of classical discrete Opial type inequalities. Some examples are presented to illustrate our results.

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