

## EXACT CONSTANTS IN ESTIMATES OF APPROXIMATION OF LIPSCHITZ CLASSES OF PERIODIC FUNCTIONS BY CESÀRO MEANS

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*Abstract.* We study the problem of computing the exact constant of approximation of classes of continuous functions by linear methods. Specifically, we describe the best constants of estimations for the rate of approximation of Lipschitz classes of periodic functions of several variables by Cesàro means of second and third order.

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### REFERENCES

- [1] V. F. BABENKO, S. V. KONAREVA, *Jackson-Stechkin-Type Inequalities for the Approximation of Elements of Hilbert Spaces*, Ukr. Math. J. **70**, (2019), 1331–1344.
- [2] V. P. BUGAETS, V. T. MARTYNYUK, *Exact constants of approximation of continuous functions by Jackson integrals*, Ukr. Math. J. **26**, (1974), 357–364.
- [3] V. P. BUGAETS, V. T. MARTYNYUK, *Exact constant for approximation of continuous functions by summation operators of Jackson type*, Ukr. Math. J. **29**, (1977), 586–590.
- [4] J. BUSTAMANTE, *Direct and strong converse inequalities for approximation with Fejér means*, Demonstr. Math. **53**, no. 1, (2020), 80–85.
- [5] A. N. DAVIDCHIK, A. A. LIGUN, A. N. MINARCHENKO, *On exact constants in approximation by some positive operators*, Dep. in VINITI 16.03.81, no. 1168–81.
- [6] V. G. DORONIN, A. A. LIGUN, *Exact constants in inequalities of the Jackson type for quadrature formulas*, Ukr. Math. J. **52**, (2000), 48–54.
- [7] L. P. FALALEEV, *On exact constants for matrix summation methods*, Sib. Math. J. **36**, (1995), 800–806.
- [8] S. FOUCART, Y. KRYAKIN, A. SHADRIN, *On the Exact Constant in the Jackson-Stechkin Inequality for the Uniform Metric*, Constr. Approx. **29**, (2009), 157–179.
- [9] V. T. GAVRILYUK, *Approximation of continuous periodic functions of one or two variables by Rogozinski polynomials of interpolation type*, Ukr. Math. J. **25**, (1973), 530–537.
- [10] A. S. B. HOLLAND, *A Survey of Degree of Approximation of Continuous Functions*, SIAM Review **23** (3), (1981), 344–379.
- [11] V. T. MARTYNYUK, *Best constants for approximations of periodic functions by Fejér operators*, Ukr. Math. J. **42**, (1990), 66–74.
- [12] I. P. NATANSON, *Constructive theory of functions*, M.: Gostekhizdat, 1949.
- [13] S. A. PICHUGOV, *Exact constants in Jackson inequalities for periodic differentiable functions in the space  $L_\infty$* , Math. Notes **96**, (2014), 261–267.
- [14] O. G. ROVENSKA, *An exact constant on the estimation of the approximation of classes of periodic functions of two variables by Cesàro means*, Matematychni Studii **57** (1), (2022), 3–9.
- [15] O. G. ROVENSKAYA, *A Sharp Constant in the Estimation of the Error of the Approximation of Classes of Differentiable Functions by the Second-Order Cesàro Means*, Sib. Adv. Math. **32**, (2022), 229–235.
- [16] G. I. RYZHANKOVA, *Approximation of differentiable functions by Cesàro sums*, Sib. Math. J. **17**, (1976), 181–184.
- [17] F. SCHURER, F. STEUTEL, *On the Degree of Approximation by the Operators of de la Vallée Poussin*, Monatshefte für Mathematik **87**, (1979), 53–64.

- [18] F. SCHURER, F. STEUTEL, *On the Degree of Approximation of Functions in  $C_{2\pi}^1$  with Operators of the Jackson Type*, J. of Approx. Theory **27**, (1979), 153–178.
- [19] M. S. SHABOZOV, G. A. YUSUPOV, *Exact constants in Jackson-type inequalities and exact values of the widths of some classes of functions in  $L_2$* , Sib. Math. J. **52**, (2011), 1124–1136.
- [20] S. B. STECHKIN, *The approximation of continuous periodic functions by Favard sums*, Proc. Steklov Inst. Math. **109**, (1971), 28–38.
- [21] S. B. VAKARCHUK, *Exact Constants in Jackson-type Inequalities and Exact Values of Widths*, Math. Notes **78**, (2005), 735–739.
- [22] S. B. VAKARCHUK, *Jackson-Type Inequalities with Generalized Modulus of Continuity and Exact Values of the  $n$ -Widths for the Classes of  $(\psi, \beta)$ -Differentiable Functions in  $L_2$ , II*, Ukr. Math. J. **68**, (2017), 1165–1183.
- [23] O. L. VINOGRADOV, *Sharp Constant in a Jackson-Type Inequality for Approximation by Positive Linear Operators*, J. Math. Sci. **107**, (2001), 3987–4001.
- [24] WANG XING-HUA, *The exact constant of approximation of continuous functions by the Jackson singular integral*, Acta Math. Sinica **14** (2), (1964), 231–237.
- [25] A. ZIGMUND, *Trigonometric series*, vol. 1, Cambridge: At the University Press, 1959.