OLD AND NEW ON THE 3–CONVEX FUNCTIONS

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Abstract. This paper is aimed to review the current knowledge of 3-convex functions and to put some of the known results into a more general context. Several open questions are also raised.

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