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VILENKIN–FOURIER SERIES IN VARIABLE LEBESGUE SPACES

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Abstract. Let $S_n f$ be the nth partial sum of the Vilenkin-Fourier series of $f \in L^1(G)$. For $1 < p_- \leqslant p_+ < \infty$, we characterize all exponent $p(\cdot)$ such that if $f \in L^{p(\cdot)}(G)$, $S_n f$ converges to f in $L^{p(\cdot)}(G)$.

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