

ON THE DEMOCRACY INEQUALITY FOR HAAR SYSTEMS AND SOME GEOMETRIC AND ANALYTICAL PROPERTIES IN HERZ SPACES ON DYADIC SETTINGS

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Abstract. In this paper we explore geometric condition to obtain that the Democracy inequality of Haar systems on Herz spaces implies that these spaces are Lebesgue spaces in the setting of spaces of homogeneous type. For this purpose, we give previously a construction of dyadic Herz spaces and prove some analytic properties.

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