Mathematical Inequalities & Applications
Volume 26, Number 4 (2023), 1021–1038
doi:10.7153/mia-2023-26-63

NEW REVERSE HÖLDER–TYPE INEQUALITIES AND APPLICATIONS

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Abstract. In this paper, we establish several Hölder-type inequalities using Jensen-type and Young-type inequalities as key tools. Particularly noteworthy is a reverse Hölder inequality with the Specht's ratio. Furthermore, we obtain a reverse Young-type inequality and we apply these results to the fractional context, both globally and locally.

Mathematics subject classification (2020): 26A33, 26A51, 26D15.
Keywords and phrases: Hölder inequality, Hölder-type inequalities, Riemann-Liouville integral operators, conformable fractional integral operators.

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Paper MIA-26-63

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